18.06 Professor Edelman Quiz 2 November 7, 2012

Grading

1

2

3

4

Your PRINTED name is: ____________________

Please circle your recitation:

1	T 9	2-132	Andrey Grinshpun	2-349	3-7578	agrinshp
2	T 10	2-132	Rosalie Belanger-Rioux	2-331	3-5029	robr
3	T 10	2-146	Andrey Grinshpun	2-349	3-7578	agrinshp
4	T 11	2-132	Rosalie Belanger-Rioux	2-331	3-5029	robr
5	T 12	2-132	Geoffroy Horel	2-490	3-4094	ghorel
6	T 1	2-132	Tiankai Liu	2-491	3-4091	tiankai
7	T 2	2-132	Tiankai Liu	2-491	3-4091	tiankai

1 (27 pts.)

P is any $n \times n$ Projection Matrix. Compute the ranks of A,B, and C below. Your method must be visibly correct for every such P, not just one example.

a) (8 pts.) $A = (I - P)P$.

b) (10 pts.) $B = (I - P) - P$. (Hint: Squaring B might be helpful.)

c) (9 pts.) $C = (I - P)^{2012} + P^{2012}$.

2 (22 pts.)

Consider a 4×4 matrix

$$A = \begin{pmatrix} 0 & x & y & z \\ x & 1 & 0 & 0 \\ y & 0 & 1 & 0 \\ z & 0 & 0 & 1 \end{pmatrix}.$$

a) (17 pts.) Compute $|A|$, the determinant of A, in simplest form.

b) (5 pts.) For what values of x, y, z is A singular?

This page intentionally blank.

3 (22 pts.)

The 3×3 matrix $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ has QR decomposition

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = Q \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ 0 & r_{22} & r_{23} \\ 0 & 0 & r_{33} \end{pmatrix}.$$

a) (7 pts.) What is r_{11} in terms of the variables $a, b, c, d, e, f, g, h, i$? (but not any of the elements of Q.)

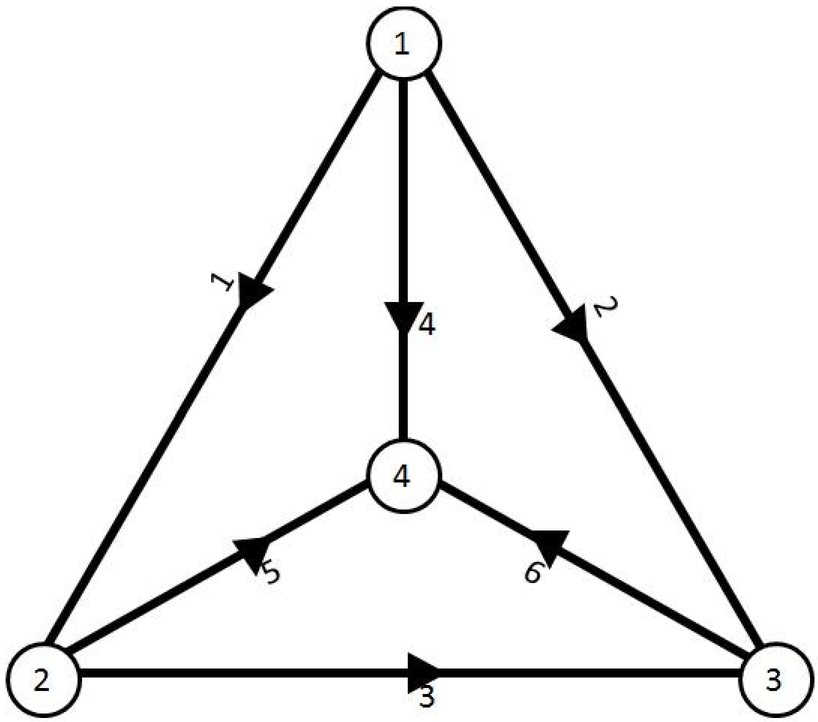
a) (15 pts.) Solve for x in the equation,

$$Q^T x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},$$

expressing your answer possibly in terms of r_{11}, r_{22}, r_{33} and the variables $a, b, c, d, e, f, g, h, i$, (but not any of the elements of Q.)

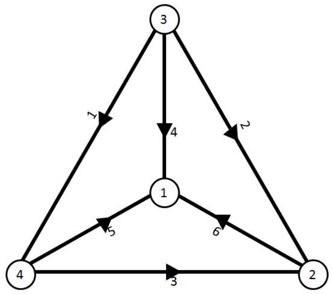
This page intentionally blank.

4 (29 pts.)



a) (15 pts.) Use loops or otherwise to find a basis for the left nullspace of the incidence matrix A for the graph above. We will start you off, one basis vector is $\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}$.

This page intentionally blank.



There are 24 ways to relabel the four nodes in the graph in part(a). Edge labels remain unchanged. One of the 24 ways is pictured above. This produces 24 incidence matrices A.

b) (7 pts.) Is the row space of A independent of the labeling? Argue convincingly either way.

c) (7 pts.) Is the column space of A independent of the labeling? Argue convincingly either way.

This page intentionally blank.